Probability, Fourth Edition

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Solutions to practice questions – Chapter 1

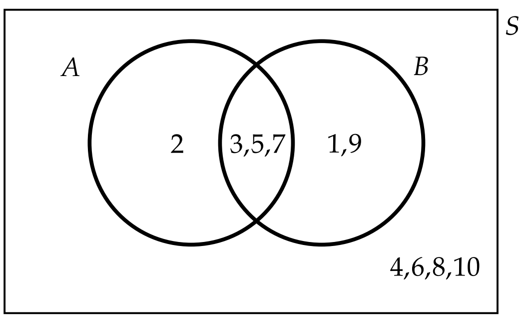
Solution 1.1

(i) $A \cap B = \{3,5,7\}$

(ii) $B' = \{2,4,6,8,10\} \quad \Rightarrow \quad A \cup B' = \{2,3,4,5,6,7,8,10\}$

(iii) $A \cup B = \{1,2,3,5,7,9\} \quad \Rightarrow \quad (A \cup B)' = \{4,6,8,10\}$

Solution 1.2



Solution 1.3

We have:

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B) = 0.4 + 0.3 - 0.15 = 0.55$$

Solution 1.4

We have:

$$\Pr(A \cup B) = 1 - \Pr((A \cup B)')$$
$$\Rightarrow 1.5\Pr(A) = 1 - \Pr(A)$$
$$\Rightarrow \Pr(A) = 0.4$$

Solution 1.5

By considering a Venn diagram, or otherwise, we have:

$$\Pr(A)+\Pr(B\cap A')=\Pr(A\cup B)=0.8$$

and since (from the question) $\Pr(A)=\Pr(B\cap A')$, we have:

$$\Pr(A)=0.4 \qquad \text{and} \qquad \Pr(B\cap A')=0.4$$

The required probability is:

$$\Pr(B')=1-\Pr(B)=1-\left[\Pr(A\cup B)-\Pr(A)+\Pr(A\cap B)\right]$$
$$=1-[0.8-0.4+0.1]=0.5$$

Solution 1.6

By the law of total probability (Theorem 1.5), we have:

$$\Pr(B)=\Pr(B\cap A_1)+\Pr(B\cap A_2)+\Pr(B\cap A_3)$$
$$\Rightarrow \quad \Pr(B\cap A_1)=\Pr(B)-\Pr(B\cap A_2)-\Pr(B\cap A_3)$$

From the question:

$$\Pr(B)=1-\Pr(B')=1-0.3=0.7$$

and:

$$\Pr(A_3\cap B)=2\times\Pr(A_2\cap B)=4\times\Pr(A_1\cap B)$$

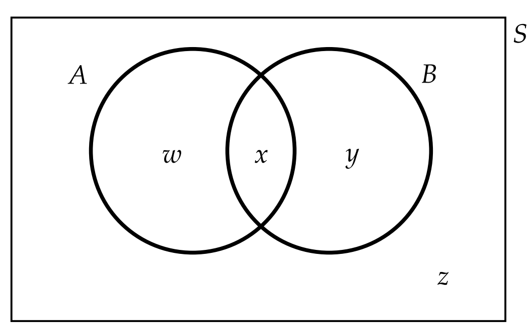
So, we have:

$$\Pr(B\cap A_1)=\Pr(B)-\Pr(B\cap A_2)-\Pr(B\cap A_3)$$
$$=0.7-2\times\Pr(B\cap A_1)-4\times\Pr(B\cap A_1)$$
$$\Rightarrow \Pr(B\cap A_1)=0.1$$

Solution 1.7

Let's solve this one graphically, using a Venn diagram.

Using the information in the question:



$$\Pr(A\cup B)=0.7$$
$$\Rightarrow w+x+y=0.7 \ \Rightarrow z=0.3$$

and

$$\Pr(A\cup B')=0.9$$
$$\Rightarrow w+x+z=0.9 \ \Rightarrow y=0.1$$

So:

$$\Pr(A)=w+x=1-y-z=1-0.1-0.3=0.6$$

Solution 1.8

Let $A = \{\text{Claims on homeowners policy}\}$ and $B = \{\text{Claims on automobile policy}\}$.

From the question we have:

$$\Pr(A) = 0.46, \quad \Pr(B) = 0.32, \quad \Pr((A \cup B)') = 0.52$$

Hence:

$$\Pr(A \cup B) = 1 - \Pr((A \cup B)') = 1 - 0.52 = 0.48$$

The probability that a claim is made on both policies is:

$$\Pr(A \cap B) = \Pr(A) + \Pr(B) - \Pr(A \cup B) = 0.46 + 0.32 - 0.48 = 0.30$$

and the probability that a claim is made on only the homeowners policy is:

$$\Pr(A \cap B') = \Pr(A) - \Pr(A \cap B) = 0.46 - 0.30 = 0.16$$

Solution 1.9

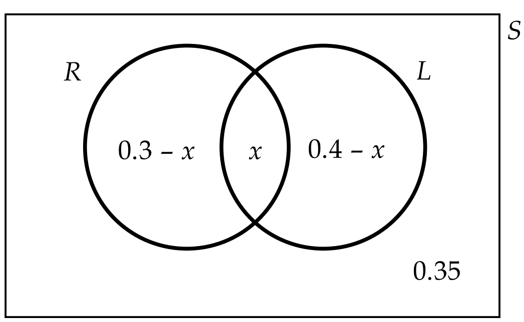
Let R correspond to referral to a specialist and let L correspond to lab work being ordered. We are given:

$$\Pr(R' \cap L') = \Pr((R \cup L)') = 0.35$$
$$\Pr(R) = 0.3$$
$$\Pr(L) = 0.4$$

Mathematically, we have:

$$\Pr((R \cup L)') = 1 - \Pr(R \cup L) = 1 - \left[\Pr(R) + \Pr(L) - \Pr(R \cap L)\right]$$
$$\Rightarrow \quad 0.35 = 1 - \left[0.3 + 0.4 - \Pr(R \cap L)\right]$$
$$\Rightarrow \quad \Pr(R \cap L) = 0.05$$

We can also solve this problem using a Venn diagram. Let $x = \Pr(R \cap L)$. The Venn diagram is:



Since the probabilities (the 4 areas in the Venn diagram) must add to 1, we have:

$$1 = 0.35 + (0.4 - x) + (0.3 - x) + x = 1.05 - x$$
$$\Rightarrow \quad x = 0.05$$

Solution 1.10

Let D, M, and O represent dental, other medical, and optical care respectively.

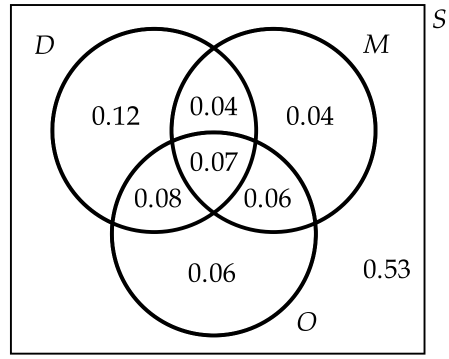
From the question, $\Pr(D \cap M \cap O) = 0.07$.

Since $\Pr(D \cap O) = 0.15$, we have:

$$\begin{aligned}\Pr(D \cap O \cap M') &= \Pr(D \cap O) - \Pr(D \cap M \cap O) \\ &= 0.15 - 0.07 = 0.08\end{aligned}$$

Similarly, since $\Pr(D \cap M) = 0.11$, we have:

$$\begin{aligned}\Pr(D \cap M \cap O') &= \Pr(D \cap M) - \Pr(D \cap M \cap O) \\ &= 0.11 - 0.07 = 0.04\end{aligned}$$



The other probabilities can be calculated in a similar fashion.

From the diagram, we can see that the required probability is 0.53.

Solution 1.11

Let H, L, and N correspond to high, low and normal blood pressure respectively, and let R and I denote regular and irregular heartbeats respectively. All the patients must fall into one of the following six (exhaustive and mutually exclusive) groupings:

$$(H \cap R), (N \cap R), (L \cap R), (H \cap I), (N \cap I), \text{ and } (L \cap I)$$

From items (i), (ii), (iii):

$$\Pr(H) = 0.14,\ \Pr(L) = 0.22 \Rightarrow \Pr(N) = 1 - \Pr(H) - \Pr(L) = 0.64$$
$$\Pr(I) = 0.15 \Rightarrow \Pr(R) = 1 - \Pr(I) = 0.85$$

From item (iv):

$$\Pr(H \cap I) = \frac{1}{3} \times \Pr(I) = \frac{1}{3} \times 0.15 = 0.05$$
$$\Rightarrow \Pr(H \cap R) = \Pr(H) - \Pr(H \cap I) = 0.14 - 0.05 = 0.09$$

From item (v):

$$\Pr(N \cap I) = \frac{1}{8} \times \Pr(N) = \frac{1}{8} \times 0.64 = 0.08$$
$$\Rightarrow \Pr(N \cap R) = \Pr(N) - \Pr(N \cap I) = 0.56$$

Finally:

$$\Pr(L \cap R) = \Pr(R) - \Pr(H \cap R) - \Pr(N \cap R) = 0.85 - 0.09 - 0.56 = 0.20$$

Solution 1.12

Of the 15 balls, there are 5 which are yellow or red and with an even number:

Red 2, Red 4, Yellow 2, Yellow 4, Yellow 6

Hence, the required probability is $\frac{5}{15} = \frac{1}{3}$.

Solution 1.13

There are $2,400 + 3,150 = 5,550$ policyholders in total, of which $0.46 \times 2,400 + 0.48 \times 3,150 = 2,616$ are female, and $5,550 - 2,616 = 2,934$ are male.

Hence, the probability that a randomly selected policyholder is male is:

$$\frac{2,934}{5,550} = 0.5286$$

Solution 1.14

(i) $\Pr(\text{Die} = 4 \cap \text{Card} = \text{Heart}) = \Pr(\text{Die} = 4)\Pr(\text{Card} = \text{Heart}) = \frac{1}{6} \times \frac{13}{52} = \frac{1}{24}$

(ii)
$$\begin{aligned}\Pr(\text{Die} \neq 6 \cap \text{Card} \neq \text{Spade}) &= \Pr(\text{Die} \neq 6)\Pr(\text{Card} \neq \text{Spade}) \\ &= \left(1 - \Pr(\text{Die} = 6)\right)\left(1 - \Pr(\text{Card} = \text{Spade})\right) \\ &= \left(1 - \frac{1}{6}\right)\left(1 - \frac{13}{52}\right) = \frac{5}{6} \times \frac{39}{52} = \frac{5}{8}\end{aligned}$$

(iii)
$$\begin{aligned}\Pr(\text{Die} < 4 \cup \text{Card} = \text{Red}) &= \Pr(\text{Die} < 4) + \Pr(\text{Card} = \text{Red}) - \Pr(\text{Die} < 4 \cap \text{Card} = \text{Red}) \\ &= \Pr(\text{Die} < 4) + \Pr(\text{Card} = \text{Red}) - \Pr(\text{Die} < 4)\Pr(\text{Card} = \text{Red}) \\ &= \frac{3}{6} + \frac{26}{52} - \frac{3}{6} \times \frac{26}{52} = \frac{3}{4}\end{aligned}$$

Solution 1.15

Let event R denote "both cars are red", let event B denote "both cars are blue", and let event W denote "both cars are white".

We want to calculate:

$$\begin{aligned}\Pr\left((R \cup B \cup W)'\right) &= 1 - \Pr(R \cup B \cup W) \\ &= 1 - \left[\Pr(R) + \Pr(B) + \Pr(W)\right] \\ &= 1 - \left[\tfrac{10}{20} \times \tfrac{9}{18} + \tfrac{6}{20} \times \tfrac{2}{18} + \tfrac{4}{20} \times \tfrac{7}{18}\right] \\ &= 1 - 0.36 = 0.64\end{aligned}$$

Solution 1.16

Since events A and B are independent, we have:

$$\Pr(A \cap B) = \Pr(A)\Pr(B) = 0.3 \times 0.4 = 0.12$$

The required probability is:

$$\Pr(A \cap B') = \Pr(A) - \Pr(A \cap B) = 0.3 - 0.12 = 0.18$$

Solution 1.17

The probability of at least one six in n rolls is:

$$1 - \Pr(\text{No sixes in } n \text{ rolls}) = 1 - \left(\frac{5}{6}\right)^n$$

So, we have:

$$1 - \left(\frac{5}{6}\right)^n > 0.95 \quad \Rightarrow \quad \left(\frac{5}{6}\right)^n < 0.05$$

$$\Rightarrow \quad n \log\left(\frac{5}{6}\right) < \log(0.05)$$

$$\Rightarrow \quad n > 16.43$$

So, the smallest value of n (which must be an integer) is 17.

Solution 1.18

The required probability is:

$$\begin{aligned}
\Pr(\text{At least one ball is blue}) &= 1 - \Pr(\text{Neither ball is blue}) \\
&= 1 - \Pr(\text{Both balls are red}) \\
&= 1 - \Pr(\text{Ball from urn 1 is red})\Pr(\text{Ball from urn 2 is red}) \\
&= 1 - \frac{6}{10} \times \frac{5}{10} = 0.7
\end{aligned}$$

Solution 1.19

The probability that all three copiers break down in a particular day is:

$$\Pr(\text{All three copiers break down}) = 0.05^3 = 0.000125$$

The probability that this event does not occur in the next 50 days is:

$$(1 - 0.000125)^{50} = 0.9938$$

So, the probability that this event does occur at least once in the next 50 days is:

$$1 - 0.9938 = 0.0062$$

Solution 1.20

We can assume that events concerning the first urn are independent of events concerning the second urn.

Let R_i denote drawing a red ball from urn i and let B_i denote drawing a blue ball from urn i. Let n be the number of blue balls in the second urn. Then:

$$\begin{aligned}
0.44 &= \Pr\left((R_1 \cap R_2) \cup (B_1 \cap B_2)\right) \\
&= \Pr(R_1 \cap R_2) + \Pr(B_1 \cap B_2) && \text{(mutually exclusive)} \\
&= \Pr(R_1)\Pr(R_2) + \Pr(B_1)\Pr(B_2) && \text{(independence)} \\
&= \frac{4}{10} \times \frac{16}{16+n} + \frac{6}{10} \times \frac{n}{16+n} \\
&= \frac{64+6n}{160+10n} \qquad \Rightarrow \quad n = 4
\end{aligned}$$

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&= \Pr(R_1)\Pr(R_2) + \Pr(B_1)\Pr(B_2) && \text{(independence)} \\
&= \frac{4}{10} \times \frac{16}{16+n} + \frac{6}{10} \times \frac{n}{16+n} \\
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\end{aligned}$$